Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2020

Subject Name: Complex Analysis

Subject Code: 5SC01COA1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 10/03/2021** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt all questions** **(07)**

a) Find the rectangular form of $z=\sqrt{3}e^{585^\circ i}$. (02)

b) Solve: $\sinh z = i$ (02)

c) Define: Analytic function with example. (02)

d) True/False: $\lim_{z\to\infty} f(z)=w_0 \Leftrightarrow \lim_{z\to 0} f\left(\frac{1}{z}\right)=w_0$ (01)

Q-2 **Attempt all questions** **(14)**

a) If $\alpha,\alpha^2,\alpha^3,\alpha^4$ are the roots of $x^5=1$ other than unity, find them and prove that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)=5$. (04)

b Show that the set of values of $Log\, i^3$ is not same as the set of values of $3Log\, i$. (03)

c) Find real and imaginary part of $\log\cos z$. (04)

d) Prove that $\tanh^{-1}x=\sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$. (03)

OR

Q-2 **Attempt all questions** **(14)**

a) Find the product of all roots of $\left(\frac{1+\sqrt{3}i}{2}\right)^{\frac{3}{4}}$. (04)

b) Solve: $z^2-(3-i)z+4-3i=0$ (03)

c) Find real and imaginary part of $\left[1+\sqrt{-3}\right]^{1+\sqrt{-3}}$. (04)

d) Find the value of $\cos^{-1}\left(\frac{3i}{2}\right)$. (03)

Q-3 **Attempt all questions** **(14)**

a) State and prove necessary condition for satisfying C-R equation for any function $f(z)$. (05)

b) Find the conjugate harmonic function $f(z)$ if the imaginary part of $f(z)$ is $\log\left(x^2+y^2\right)+x-2y$. (05)

c) Check C-R equations are satisfied are not for the function $f(z)=z^{\frac{3}{2}}$, if it satisfies hence find $f'(z)$. (04)

OR

Q-3 **Attempt all questions** **(14)**

a) State and prove chain rule for derivatives. (07)

b) If $f(z)=\begin{cases}\frac{\bar{z}^2}{z} & ; \ z\neq 0\\ 0 & ; \ z=0\end{cases}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at origin. (05)

c) Define: Continuous function and give one example of the function which is continuous but not differentiable. (02)

SECTION – II

Q-4 **Attempt all questions** **(07)**

a) State Maximum modulus principle. (02)

b) Evaluate: $\int_C \frac{e^z}{z^2-1}dz$, where C is the circle with unit radius and centre at $z=1$. (02)

c) Find the residue of $f(z)=\frac{1-e^{2z}}{z^4}$. (02)

d) Classify the singularity for $\frac{z-\sin z}{z^2}$ at $z=0$. (01)

Q-5 **Attempt all questions** **(14)**

a) State and prove Cauchy's integral formula. (05)

b) Verify Cauchy's theorem for $f(z)=z^2$ taken over the boundary of a square with vertices at $\pm1\pm i$ in counter clockwise direction. (05)

c) Find an upper bound for the absolute value of the integral $\int_C \frac{\sqrt{z}}{z^2+1}dz$ where C is the contour given by upper half of the circle $|z|=3$. (04)

OR

Q-5 **Attempt all questions** **(14)**

a) Let $P(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n \left(a_n \neq 0\right)$ be a complex valued polynomial of degree $n(n \geq 1)$ then there exist at least one complex root z_0 such that $P(z_0) = 0$. (05)

b) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-1)^2} dz \; C : |z| = 3$ by using Cauchy's integral formula. (05)

c) Integrate the function $f(z) = x^2 + iy^2$ around the ellipse C defined by $x = 2\cos t, y = 3\sin t$, where $0 \leq t \leq 2\pi$. (04)

Q-6 **Attempt all questions** **(14)**

a) State and prove Taylor's theorem. (05)

b) Find the Laurent expansions for the function $f(z) = \frac{7z-2}{z(z^2 - z - 2)}$ in the annulus $1 < |z+1| < 3$ (05)

c) Evaluate: $\int_C z^2 e^{\frac{1}{z}} dz; \; C : |z| = 1$ by using Cauchy's residue theorem. (04)

OR

Q-6 **Attempt all questions** **(14)**

a) State and prove Laurent's series. (08)

b) Expand $f(z) = \frac{z+5}{z^2 + 3z + 2}$ as Taylor's series about the point $z = 0$ upto four terms. (03)

c) Evaluate $\int_C \tan z \, dz \; C : |z| = 2$ by using Cauchy's residue theorem. (03)